Active notch filters

Design theory behind the development of discrete frequency rejection circuits

by Yishay Nezer, B.Sc.

We often need to separate a wanted signal from periodic interference. This may happen, for example, when a whistle or a power-line hum is disturbing a radio programme. In simple cases a filter which has zero transmission at one discrete frequency and unity transmission at all other frequencies is sufficient. In contrast to a practical low-pass or high-pass filter, an almost ideal notch filter can be realized with only one section; moreover it can be voltage tuned or even automatically track the interference.

The major class of notch filters, both passive and active, is of the second order and has the following transfer function:

$$G(s)=\frac{s^2+\omega_0^2}{s^2+\frac{\omega_0}{Q_0}s+\omega_0^2} \qquad (1)$$

where ω_0 is the rejection frequency and Q_0 is the figure of merit of the filter, which is given by $Q_0=\frac{\omega_0}{\Delta\omega}$ where $\Delta\omega$ is the rejection bandwidth defined by the 3dB attenuation points. Some typical frequency response curves are plotted in Fig. 1 with Q_0 as the fixed parameter.

Many passive notch networks are known. This article will deal mainly with *RC* networks, since the use of coils is inconvenient, particularly at low frequencies. The best-known *RC* notch network is the symmetric twin-tee illustrated in Fig. 2(a), for this network $\omega_0=1/RC$ and $Q_0=1/4$. The function is, of course, realized only if the network is fed by a voltage source and subjected to an infinite load.

Another well-known notch network is the Wien bridge. This network is characterized by $\omega_0=1/RC$ and $Q_0=1/3$. The left side of the bridge shown in Fig. 2(b) is composed of equal resistors and capacitors and, in order to obtain an infinite null, the other two resistors must satisfy the relationship $r_1=2r_2$. The notch response, however, can be achieved even if the corresponding components in the reactive side of the bridge are not equal. The rejection frequency will then be $\omega_0=1/\sqrt{R_1R_2C_1C_2}$ but the ratio r_1/r_2 will no longer equal two. An important special case occurs when $R_1=2R_2$ and $C_1=C_2/2$; we then have $r_1=r_2$.

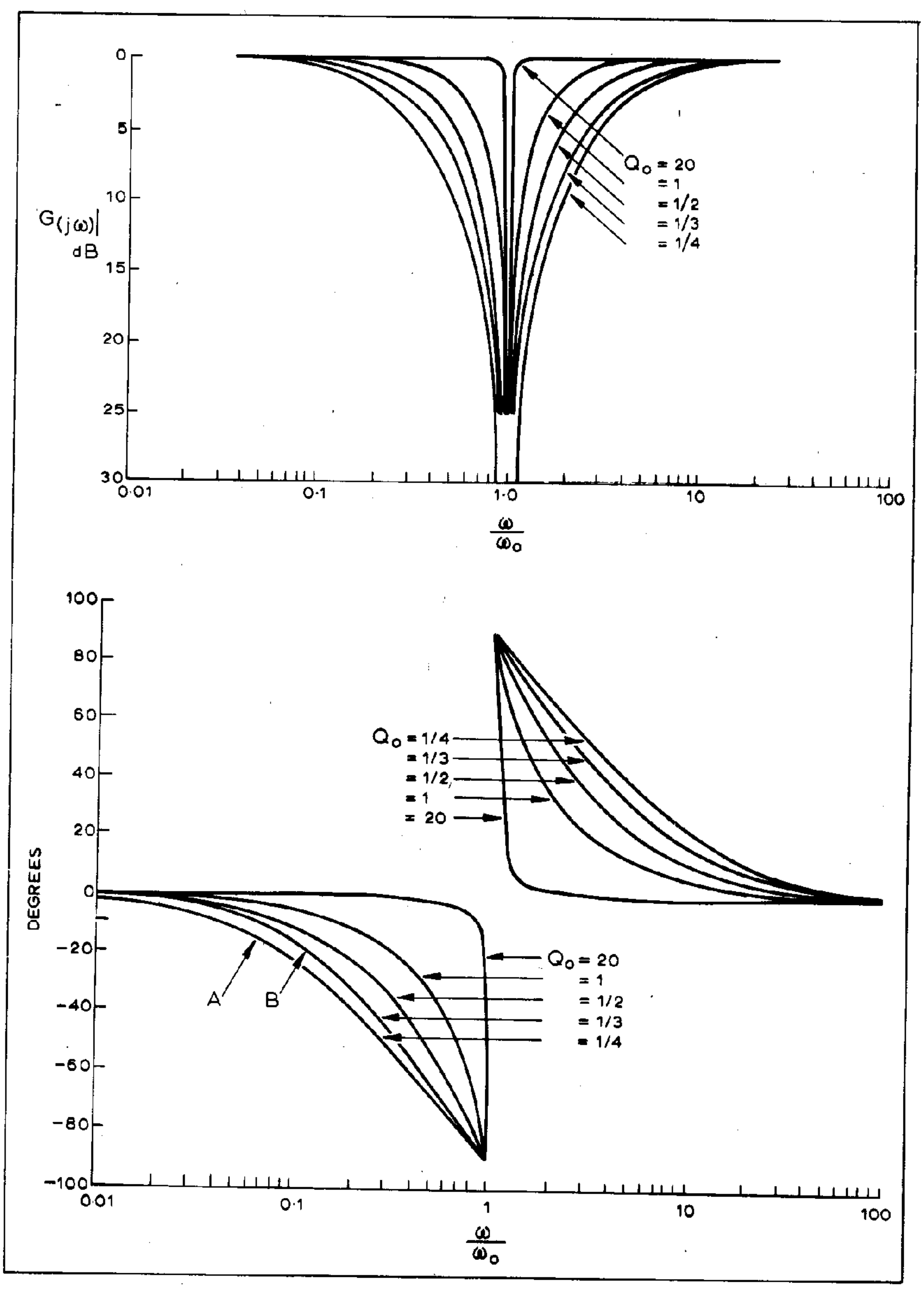


Fig. 1. Normalized phase and magnitude response curves of a notch filter for several values of Q_0.

A drawback of the above two networks is that in order to vary the centre frequency and still maintain the infinite null, two or three closely matching ganged variable components must be used. Several RC bridge networks are known in which a single component is sufficient to control the rejection frequency. However, their practical significance is limited because the frequency response becomes severely asymmetric as the rejection frequency is varied.

A more acceptable variable network was proposed by Hall[1]. It is shown in Fig. 2(c). This network can be tuned by means of a single potentiometer and the tuning law is $\omega_0 = 1/RC\sqrt{\alpha(1-\alpha)}$ which in theory spans the whole frequency range. In practice the tuning range is quite limited due to the extreme non-linear dependence of ω_0 on α. However, this network has unity gain on both sides of the null frequency, irrespective of the tuning.

However, unlike the twin-tee and the Wien bridge it is asymmetric on a logarithmic frequency scale. This follows from the fact that the transfer function of this network is not given by expression (1) but contains an additional real pole and real zero.

A similar potentiometer tuned null network based on the twin-tee was proposed by Andreyev[2].

All the networks discussed so far are characterized by low selectivity. In fact[3], no passive RC notch network, however complex, is capable of achieving Q_0 higher than 0.9. If the notch filter must be passive, a relatively high Q_0 may be achieved by including an inductance as in the bridged-tee network shown in Fig. 3. In order to achieve a complete null this network must satisfy the two conditions:

$$\omega_0^2 = C_1 + C_2/LC_1C_2 \text{ and}$$
$$\omega_0^2 = 1/rRC_1C_2 \qquad (2)$$

The figure of merit will then be $Q_0 = 2\,\omega_0 L/r$, i.e. proportional to the quality factor of the coil.

Active notch filters

As has been mentioned above, passive RC notch filters suffer from a low selectivity. A theoretically unlimited selectivity can be obtained by the use of active notch filters. These can be built by various active realizations of the transfer function given by expression (1). Simple active circuits are based on passive null networks in which the selectivity is raised by means of negative feedback.

One such scheme is shown in Fig. 4 and the effect of feedback can be explained as follows: When the feedback loop is open the network is simply a passive null network with a passband gain of A_0 represented by curve (a) in Fig. 5. When the feedback loop is closed, the network tends to maintain a voltage gain of $A_0/(1+A_0)$. However, it fails to do so where the forward gain is low, i.e. in the vicinity of ω_0. As a result, the response curve is compressed as shown in curve (b) and the rejection band is narrower. As an additional benefit, the active filter can now be cascaded without being subjected to loading.

The calculated transfer function of the active notch filter is:

$$G(s) = \frac{A_0}{1+A_0}\cdot\frac{s^2+\omega_0^2}{s^2+\omega_0 s/(1+A_0)Q_0+\omega_0^2} \qquad (3)$$

A different realization is shown in Fig. 6 which relies on a single, less-than-unity gain amplifier. It can be seen that there are two feedback paths in the configuration, a positive unity-gain feedback which renders the effective gain of the amplifier equal to $K/(1-K)$ instead of K, and a negative feedback which subtracts the output voltage from the input. If $K/(1-K) = A_0$, this method is equivalent to the former and the transfer function is:

$$G(s) = \frac{s^2+\omega_0^2}{s^2+\omega_0 s(1-K)/Q_0+\omega_0^2} \qquad (4)$$

in which the selectivity is multiplied by $1/(1-K)$.

Practical circuits

The simplest amplifier for the above method is the emitter follower. How-

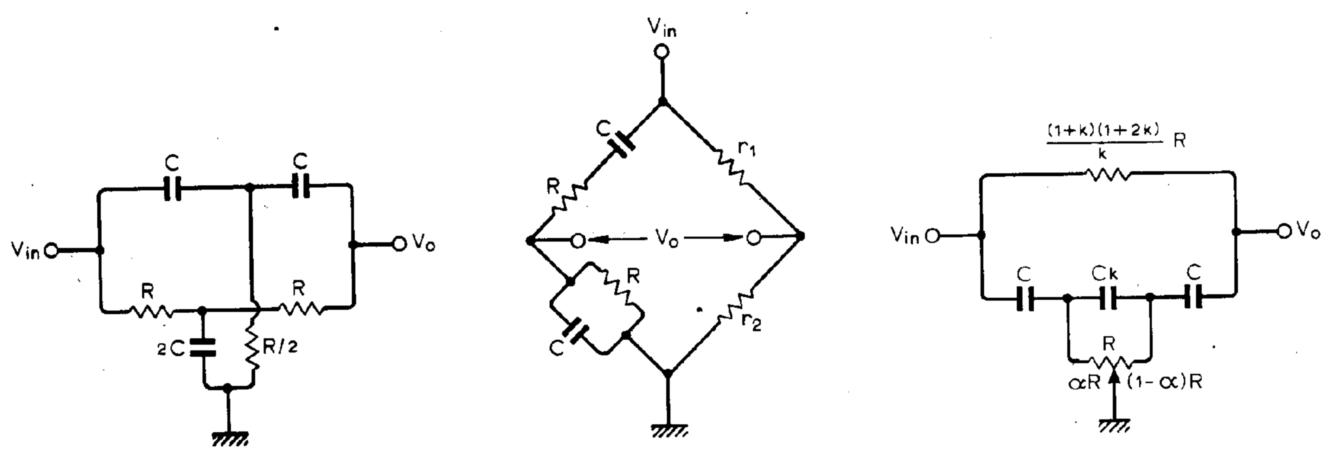


Fig. 2. Three RC null networks: (a) the symmetric twin-tee, (b) the Wien bridge, (c) a potentiometer-tuned network.

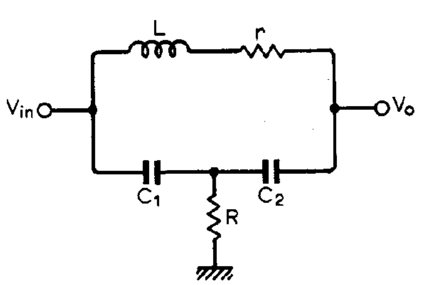


Fig. 3. Bridge-tee RCL null network, the selectivity of which depends on the Q factor of the coil.

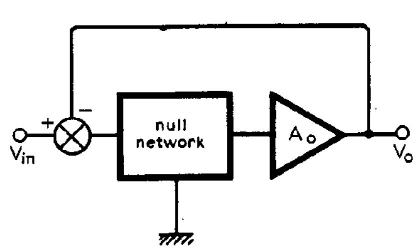


Fig. 4. Basic active configuration for enhancing the selectivity of passive notch filters.

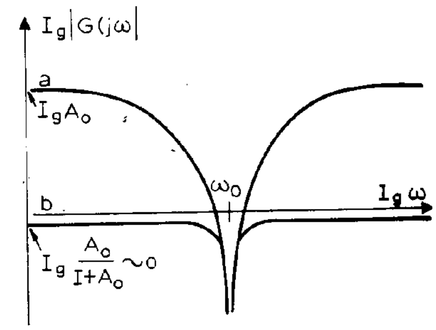


Fig. 5. Frequency characteristics of the network in Fig. 4 (a) open loop, (b) closed loop.

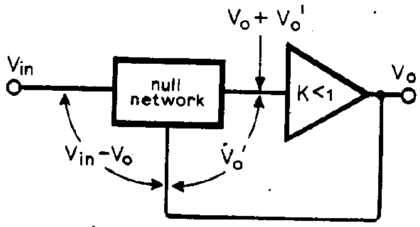


Fig. 6. Practical configuration for enhancing the selectivity of a passive notch network with a single voltage amplifier having a gain of less than unity.

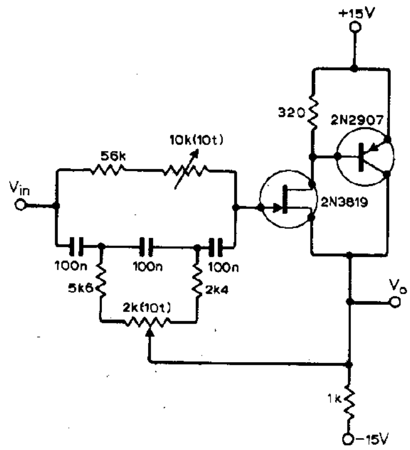
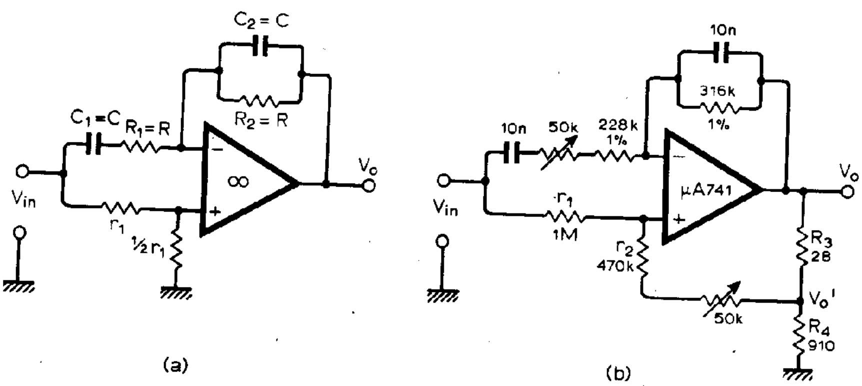


Fig. 7. Simple potentiometer-tuned active notch filter based on the network in Fig. 2(c). Tuning range 200 ± 10Hz; rejection bandwidth 10Hz (3dB).



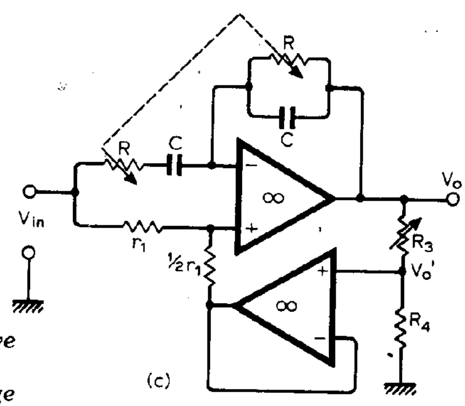


Fig. 8. (a) Wien bridge converted to a three-terminal network; (b) 50Hz active notch filter; (c) variable rejection frequency and bandwidth Wien-bridge active notch filter.

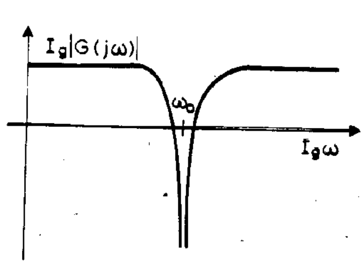


Fig. 9. Wider rejection bandwidth is attained by cascading two notch filters.

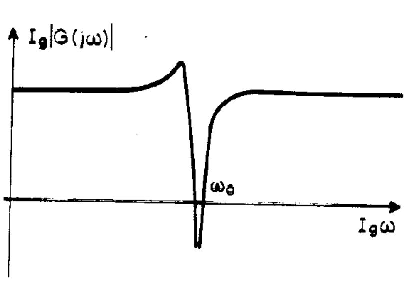


Fig. 10. Asymmetry in the frequency response curve of high Q notch filter as a result of excessive phase shift in the amplifier.

ever, it is not very suitable because in order to prevent the null network from being loaded by the relatively low-input impedance, the resistors must be relatively low and the capacitors must be correspondingly large.

It is obviously possible to replace the transistor by an f.e.t. and use smaller capacitors, but the increase in the selectivity would be limited due to the smaller gain usually associated with the f.e.t. The bootstrapped source follower benefits from high-input impedance and also a gain closer to unity and is shown in Fig. 7 together with the network of Fig. 2(c).

The networks discussed above have ideally an infinite attenuation at the notch frequency. Practically, the attention is limited by the tolerances of the components and is typically 40dB for 1% tolerance. This figure may be exceeded by trimming and is ultimately limited by stray capacitance.

The Wien bridge is attractive owing to its simplicity. However, it is not a three-terminal network, and cannot be activated directly.

The circuit in Fig. 8(a) is a Wien bridge built around an operational amplifier. In spite of being active, the factor of merit is only ½ instead of ⅓ in the passive bridge (it does not belong to either of the schemes shown in Figs. 4 and 6) yet, being a three-terminal network, its selectivity can be improved as shown in Fig. 6. Since the output impedance is already zero an additional buffer amplifier is unnecessary, so that we only have to decrease the gain to below unity by a voltage divider and close the feedback loop at r_2. The network Fig. 8(b) then contains an equivalent amplifier whose gain and output impedance are:
$K = R_4/(R_3 + R_4)$ and $R_3 \| R_4$ respectively. Accordingly, the latter value must be subtracted from r_2 or must be much smaller. Alternatively the voltage divider can be buffered as in Fig. 8(c).

If we use V_0 as the output of the filter instead of V'_0, an advantage results with respect to the network of Fig. 6, in that the passband gain is unity instead of K. However, as the internal amplifier's gain is $K = R_4/R_3 \times R_4$, the factor of merit will be

$$Q = Q_0/(1-K) = (1 + R_4/R_3)/2 \qquad (5)$$

and the rejection bandwidth can be varied by means of either R_3 or R_4. The null frequency can be varied, for example, by R_1 and the notch depth can then be adjusted by means of r_2.

If it is desired to vary the rejection frequency over a wide range, it is best to vary simultaneously resistors R_1 and R_2. If the tracking is good, the null will be maintained throughout the tuning range without further adjustments.

High Q notch filters

At a frequency $\omega = \omega_0(1 \pm \epsilon)$ close to the resonant frequency, that is $\epsilon \ll 1/2Q$, the response of a notch filter can be approximated by two

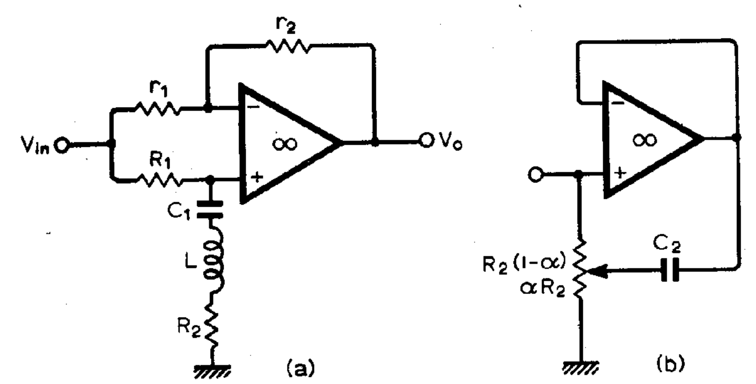


Fig. 11. (a) An active RCL notch filter; (b) a potentiometer-controlled simulated grounded coil.

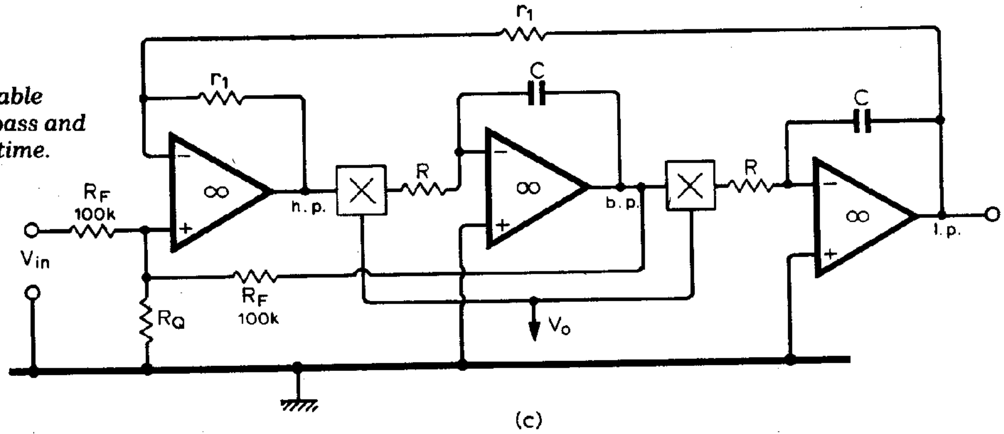


Fig. 12. Programmed state variable filter provides low-pass, band-pass and high-pass outputs at the same time.

straight lines with slopes ±2Q, and the filter can be used as a frequency discriminator. If Q is large, very small frequency deviations can be observed. On the other hand, if a high Q notch filter is used to reject a power-line hum, for example, a slight deviation of the frequency from its nominal value will suffice to render the attenuation excessively low. In this case a filter with an infinite attenuation over a *band* of frequencies would be desirable. However, such a filter cannot be realized. It is then possible either to lower the Q of the filter or to stagger-tune two or more filters in cascade and obtain a frequency response as in Fig. 9. A more elaborate solution is to use an interference tracking notch filter as suggested in the last paragraph.

When dealing with high Q notch filters, the increasing sensitivity of the notch symmetry and depth to the tolerance of the passive components becomes a serious problem. A practical solution is to use stable capacitors and trim resistors for the required notch frequency and depth. It has been seen that the most suitable network from this standpoint is the Wien bridge, but for ultimate stability the state variable filter (see below) is required owing to its extreme stability.

Another problem in realizing high Q notch filters is the roll-off in the open loop gain of operational amplifiers at high frequencies. It is found that there is a limitation on the maximum possible Q which is inversely proportional to the rejection frequency. This limitation may cause asymmetric frequency response (Fig. 10), even if the values of the passive components are accurate. With the increase in Q, the size of the "the hump" increases to the appearance of oscillations. If the notch frequency is preset this can be rectified by adding an RC phase leading section at the input of the amplifier of Fig. 6 and experimentally adjusting the time constant. If the notch frequency must be variable, the only remedy is to use a wide bandwidth amplifier. However, at fairly high frequencies high Q inductors are available, and a passive filter such as the one in Fig. 3 may be preferable.

Simulated inductance

The circuit of Fig. 11(a) is an active bridge similar to that shown in Fig. 8(a), but has a series resonant circuit in one of its arms. If $R_1/R_2=r_1/r_2$, the circuit will behave as a notch filter whose rejection frequency and factor of merit are

$$\omega_0=1/\sqrt{LC_1} \text{ and } Q_0=\omega_0 L/R_2 \quad (6)$$

In order to avoid using an inductance, the series connection of R_2 and L can be replaced by the circuit of Fig. 11(b)[4] which is equivalent to a coil whose value is $L=C_2R_2\alpha(1-\alpha)$ in series with a resistor R_2. The resulting notch filter has a rejection frequency and factor of merit given by:

$$\omega_0=1\sqrt{C_1C_2R^2{}_2\alpha(1-\alpha)}$$
$$\text{and } Q_0=\alpha(1-\alpha)\sqrt{C_2/C_1} \quad (7)$$

respectively; the tuning law is thus exactly the same as that of Fig. 2(c).

This network can also be tuned by means of C_1, which may consist of a small trimming capacitor if the simulated inductance is made appropriately large.

State variable filters

A unique active notch filter may be realized by the so-called state variable method[5]. This method is based on a multiple feedback network containing integrators and adders. In spite of the rather large number of operational amplifiers, the number of capacitors needed for the realization of any arbitrary transfer function is minimal.

The basic building block shown in Fig. 12 simultaneously provides three transfer functions of the second order; these are high-pass, band-pass and low-pass function, given by:

$$V_H(s)=Ks^2/(s^2+(\omega_0/Q_0)s+\omega^2{}_0) \quad (8)$$

$$V_B(s)=K\frac{\frac{-1}{\tau_1}s}{s^2+\frac{\omega_0}{Q_0}s+\omega_0^2}$$

$$V_L(s)=K\frac{\omega_0^2}{s^2+\frac{\omega_0}{Q_0}s+\omega_0^2}$$

The resonant frequency ω_0 is determined by the time constants τ_1, τ_2 of the integrators, and is given by $\omega_0=1/\sqrt{\tau_1\tau_2}$. The factor of merit is $Q_0=1/K\sqrt{\tau_2/\tau_1}$ where $K=1/(1+R_F/2R_Q)$.

This configuration does not provide complex zeroes and in order to obtain the symmetric notch response given by expression (1) we must sum the high-pass and low-pass outputs. The filter obtained contains four operational amplifiers but has the following characteristics: if τ_2 varies, the rejection frequency is varied while the bandwidth $\Delta\omega = \omega_0/Q_0$ remains unchanged; if τ_1 and τ_2 vary simultaneously, the rejection frequency varies linearly while the factor of merit Q_0 remains unchanged; if K is varied with the aid of the resistor R_Q, the rejection frequency remains unchanged and the rejection bandwidth and gain alone will change; the network is quite insensitive both to the values of the passive components and to the gain of the amplifier. Its stability approaches that of passive filters.

The transfer function of a conventional integrator is: $G(s)=1/\tau s=1/RCs$ and the variation of τ can be obtained by varying R or C. If we connect an amplifier with a gain K in series with the integrator, the transfer function changes to $G(s)=K/\tau s$; i.e., τ is decreased without altering R or C. If an analogue multiplier is substituted for the amplifier, an integrator is obtained, in which the time constant τ is dependent on a control voltage. A notch filter with a constant Q and rejection frequency directly proportional to the control voltage can thus be built from two such integrators.

It has already been mentioned that the maximum Q factor which can be obtained in an active filter realization is limited – for a given resonant frequency – by the bandwidth of the operational amplifier. The state variable method is no exception to this rule. Design considerations resulting from these limitations were discussed by Thomas[6].

Bandpass filter synthesis

A common disadvantage which is probably shared by all accepted active bandpass realizations is that the bandwidth is inversely proportional to the midband gain; in other words an increase of Q is accompanied by a proportional rise in gain, which is often undesirable.

A different realization of bandpass filter is given in Fig. 13(a) based on the equation:

$$A\left(1-\frac{s^2+\omega_0^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}\right)=A\,\frac{(s/Q).\,\omega_0}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \qquad (9)$$

which results in a bandpass transfer function whose midband gain is independent of Q. The configuration operates as follows: at frequencies which are remote from ω_0 the notch filter transmission is unity, and the input to the differential amplifier is zero. At frequency ω_0 the notch filter transmission is zero and the input to the amplifier is unity. As equation (9) shows, the resulting bandpass filter has the same Q as the notch filter. Now it is easy to realize notch filters in which the

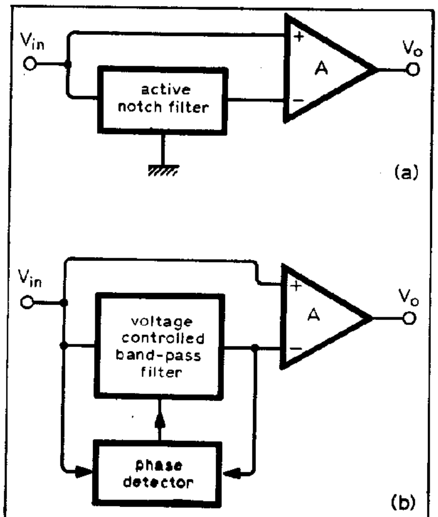


Fig. 13. (a) Realization of constant variable-bandpass filter through the use of notch filter; (b) tracking notch filter, for rejecting a drifting interference.

selectivity is controlled by a single resistor, such as the one shown in Fig. 8. If such a filter is incorporated, a constant-gain variable bandpass filter is obtained.

Interference-tracking notch filter

It has been previously mentioned that a sufficiently narrow-band notch filter may not be effective in rejecting interference which drifts in frequency. Such interference could in principle be tracked by a phase-locked loop, and the output of the loop, which is proportional to the frequency, then applied to a voltage-controlled notch filter. A drawback of such a method is that it is an open-loop system and any residual interference at the output due to imperfect tracking is not corrected for.

In contrast to the phase-locked loop, which is a signal-tracking oscillator, a signal-tracking band-pass filter can be easily built[7]. It is similar to the phase-locked loop and consists of a voltage-controlled band-pass filter, a phase detector and a low-pass filter. The closed loop then centres the band-pass filter on the signal by maintaining an (ideally) zero phase shift between the interference and the output of the filter. This bandpass filter can be converted to a signal-tracking notch filter as shown in Fig. 13(b), in which the filtered interference is subtracted from the original input. However, unlike conventional notch filters, the input to this filter must contain a minimum of interference for locking to occur.

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